Introduction to credit risk

Marco Marchioro
www.marchioro.org

December 1st, 2012

Lecture Summary

- Credit risk and z-spreads
- Risky yield curves
- Riskless yield curve
- Definition of credit derivatives. Comparison with other derivatives
- Survival and default probabilities, hazard rates
- Credit default swap, spread, and upfront quotes
- Standard definitions of default, restructuring, and seniority
- Bootstrap of probability curve from quoted CDS

Pricing a fixed-rate coupon bond

Given a bond we should be able to compute its present value just by bootstrapping the discount curve $D_{\mathsf{L}}(T)$,

$$\mathsf{PV}_{\mathsf{bond}} = \sum_{i=1}^{n} D(t_i)\, N\, C_i + D(t_n)\, N \tag{1}$$

where C_i are the coupons at the deterministic times t_i.

Setting $N{=}100$ we can compute the *dirty price* is computed as

$$P_{\mathsf{dirty}} = \mathsf{PV}_{\mathsf{bond}}(N = 100)\,, \tag{2}$$

where t_j is the start date of the current coupon and t is today

However, in the market with obtain very different price quotes!

Zero volatility spread

The zero-volatility spread, or z-spread, is the continuously compounded* spread Z that should be applied to the Libor curve in order to price a bond consistently with market quotes.

Given a fixed-rate coupon bond, we solve for its z-spread Z so that

$$\frac{P_{\text{dirty}}}{100} = \sum_{i=1}^{n} e^{-Z\,t_i} D_{\mathsf{L}}(t_i) C_i + e^{-Z\,t_n} D_{\mathsf{L}}(t_n) \tag{3}$$

where $D_{\mathsf{L}}(T)$ is the discount at time T on the Libor curve.

*Unfortunately, there is more than one standard and, sometimes, annual compounding is used

Compounding for zero-volatility spread

More often than not annual compounding is used for the z-spread

$$\frac{P_{\mathsf{dirty}}}{100} = \sum_{i=1}^{n} \frac{C_i}{(1+Z)^{t_i}} D_{\mathsf{L}}(t_i) + \frac{1}{(1+Z)^{t_n}} D_{\mathsf{L}}(t_n) \tag{4}$$

Note

- The z-spread is equivalent to the bond price
- For higher credit and short maturities z-spread could be negative

Risky discount curve

The pricing equation for a bond with credit risk

$$\mathsf{PV}_{\mathsf{bond}} = \sum_{i=1}^{n} e^{-Z\,t_i} D_{\mathsf{L}}(t_i)\,N\,C_i + e^{-Z\,t_n} D_{\mathsf{L}}(t_n)\,N \tag{5}$$

Can be written as

$$\mathsf{PV}_{\mathsf{bond}} = \sum_{i=1}^{n} D_{\mathsf{Z}}(t_i)\,N\,C_i + D_{\mathsf{Z}}(t_n)\,N \tag{6}$$

By defining the risky curve D_{Z}

$$D_{\mathsf{Z}}(t) = e^{-Z\,t} D_{\mathsf{L}}(t) \tag{7}$$

Bond seniority

Bond seniority is the priority given to the holder of a certain bond in the queue of debt repayment. Typical bond seniority's are

Secured
An underlying hard asset is associated to this type of securities

Senior
Senior debt issues have the priority in the debt queue

Junior
Junior, a.k.a. subordinate, debt will be paid after all the senior debt creditors are completely paid

Risky discount curves

- It is possible to compute the price of a risky bond assuming a discount curve given by the Libor curve and a z-spread
- For given issuer risky discount curve can be bootstrapped from quoted bond prices
- Data providers sometimes publish discount curves by rating/sectors

Computation of yield

The bond *dirty price* is defined as the bond PV for a notional of 100 units of currency,

$$P_{\text{dirty}} = \sum_{i=1}^{n} D_{\text{Z}}(t_i)\, 100\, C_i + D_{\text{Z}}(t_n)\, 100 \tag{8}$$

and can be used to infer the unique yield Y such that

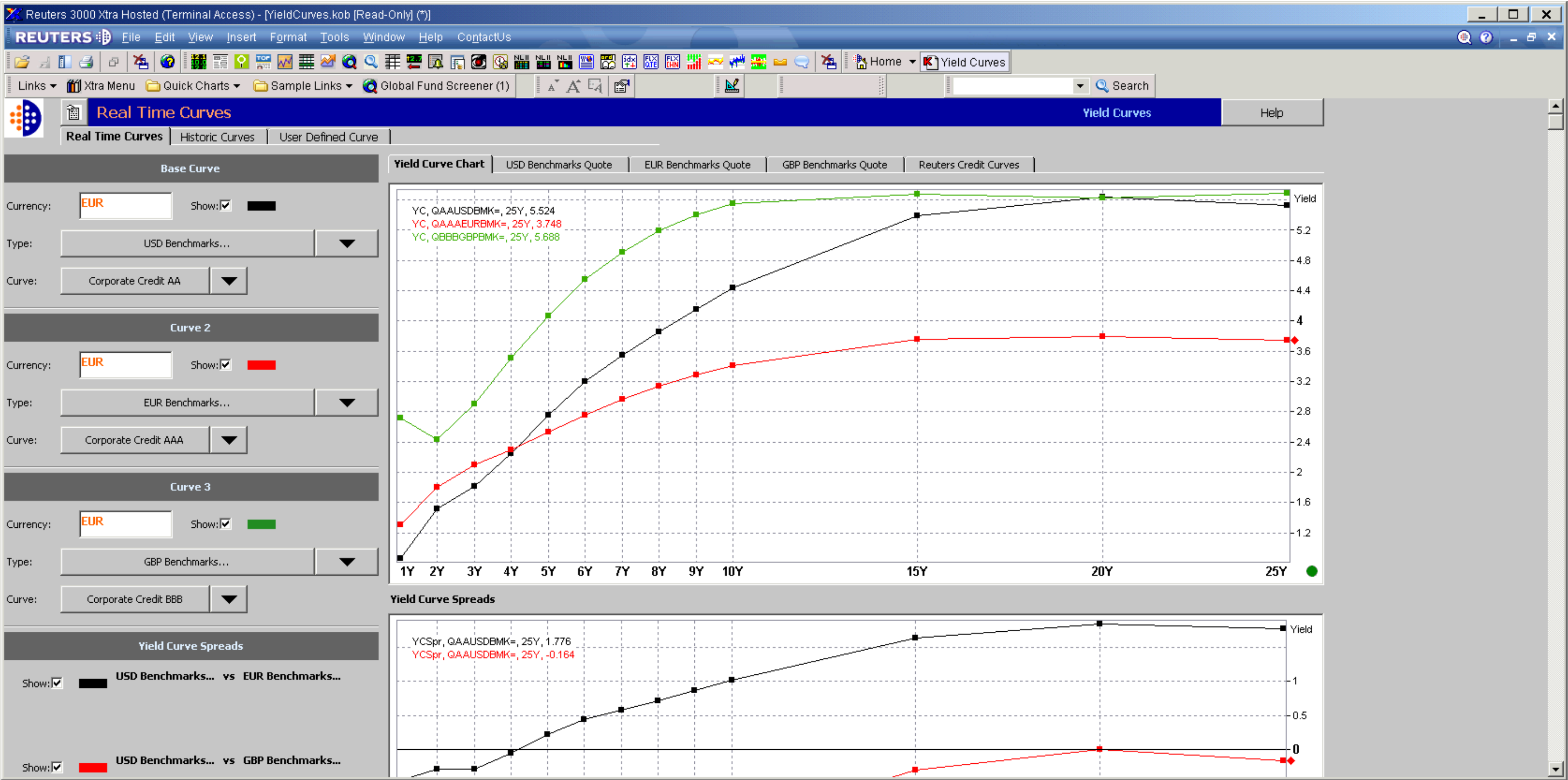
$$P_{\text{dirty}} = \sum_{i=1}^{n} \frac{100\, C_i}{(1+Y)^{t_i}} + \frac{100}{(1+Y)^{t_n}} \tag{9}$$

We have used annual compounding (the market uses usually simple compounding up to one year and yearly compounding thereafter).

Indexes of yield curves

- Similar bonds are aggregated into bond indexes, e.g. by rating, sector, issuer
- Given a portfolio of bonds we can compute its average yield at different maturities
- The average yields of major indexes are quoted on the market

Yield curves for ratings: AAA, AA, BB



Spread over reference

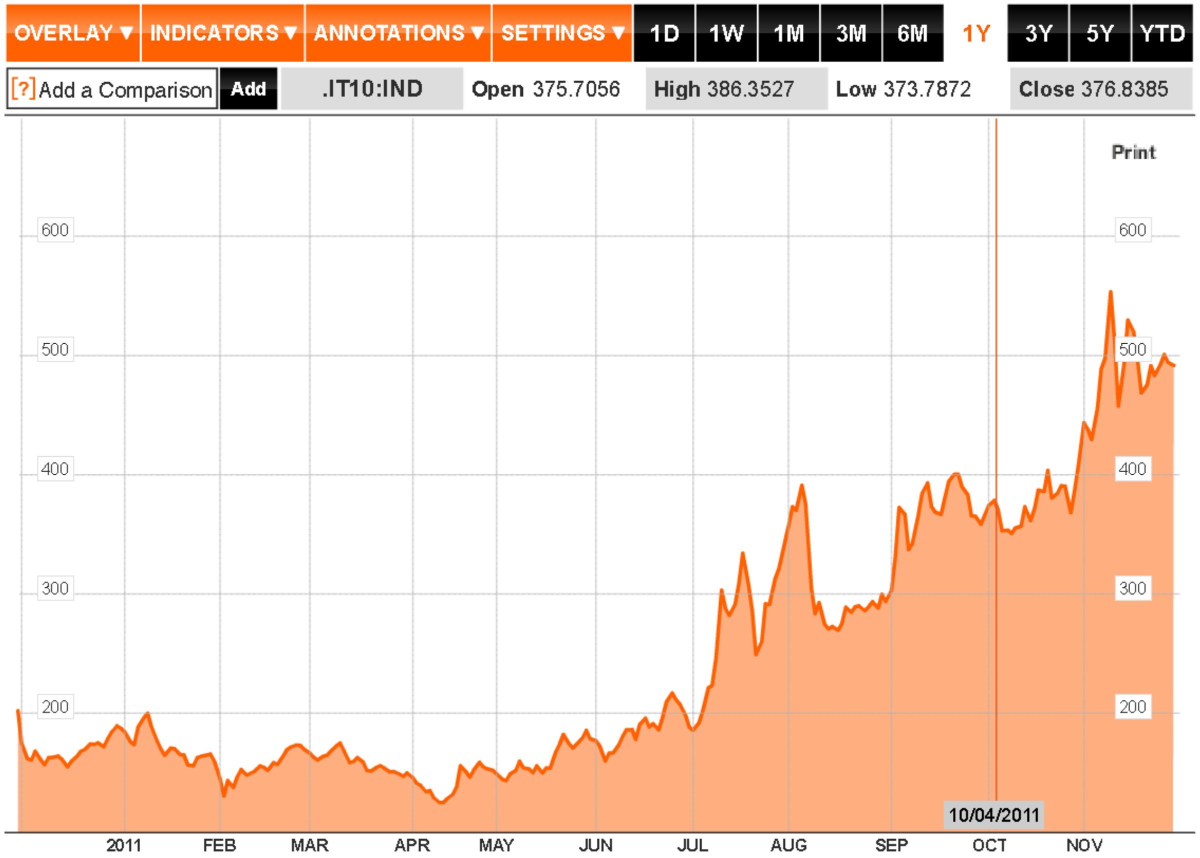
- Bond prices are usually quoted as yield
- Yields and z-spreads are equivalent to prices
- Two bond are compared by their yield
- The difference between a bond yield and the refernec yield is called *spread*

$$S_{\text{pread}} = Y_{\text{BTP}} - Y_{\text{Bund}}$$


CHE EBBREZZA
VAI MARIO! A TUTTO SPREADDD
WILLIAM WYLER
vacanze romane
PECK
HEPBURN

Lo Spread

Bund-BTP spread



What is a riskless yield curve

Now we can compute credit-risk!
One more question: what is riskless?

What should we bootstrap the riskless yield curve from?

- Treasury bonds
- IR futures and swaps
- EONIA and EONIA swaps (Libor vs EONIA with daily comp.)
- Gold swaps (Libors vs fixed rate on gold paid in gold)

This is still an open question!

Questions?

Pricing of a risky floater (1/2)

A Floating-Rate coupon Note (FRN) pays coupon proportional to the Libor fixing at the beginning of the coupon.

$$C_i = N\,\tau_i \cdot (L_i + s) \tag{10}$$

with the coupon year fraction τ_i, the Libor rate L_i, and the spread s.

Using the risky curve D_{Z} we can compute the price as

$$\mathsf{P}_{\mathsf{dirty}} = \sum_{i=1}^{n} D_{\mathsf{Z}}(t_i) \cdot \tau_i \cdot \Big[r_{\mathsf{fwd}}(T_i, T_{i+1}) + s \Big] \cdot 100 + D_{\mathsf{Z}}(t_n) \cdot 100 \tag{11}$$

Pricing of a risky floater (2/2)

Note that the forward rate r_{fwd} should be computed using the Libor $D_L(T)$ curve

$$r_{\text{fwd}}(T_i, T_{i+1}) = \frac{1}{\tau_i}\left[\frac{D_L(T_i)}{D_L(T_{i+1})} - 1\right] \tag{12}$$

The bond price depends on **two** interest-rate curves:

- The forecast curve $D_L(T)$
- The discount curve $D_Z(T)$

Pricing of an equity-linked bond

Consider an equity-linke bond with coupons

$$C_i = N\,\tau_i \cdot \mathsf{Payoff}_i(S_0, S_1, \ldots, S_i) \qquad (13)$$

with $S_0, S_1, \ldots, S_i$ the fixing of the underlying equity index.

Then the bond price can be computed as a function of the optionlet prices OP

$$\mathsf{P}_{\mathsf{dirty}} = \sum_{i=1}^{n} D_{\mathsf{Z}}(t_i) \cdot \tau_i \cdot \mathsf{OP}_i\left(S_0, \sigma, D_L(T)\right) \cdot 100 + D_{\mathsf{Z}}(t_n) \cdot 100 \qquad (14)$$

Note option prices are computed using $D_L(T)$ and assumed to pay at the end of the coupon T_i

Monte Carlo simulations for equity-linked bond

- Generate the equity paths using zero-rate of the forecast curve $D_L(T)$ and volatility σ starting from S_0
- Compute the cash flows C_i at the payment date T_i
- Discount the cash flows using the risky curve $D_Z(T)$
- Average all simulations

$$\mathsf{PV} = \frac{1}{N_j} \sum_{\text{path } j}^{N_j} \sum_{\text{future } i} D_{\mathsf{Z}}(t_i) \cdot C_i(S_0, S_1^j, \ldots, S_i^j) \qquad (15)$$

Questions?

Credit derivatives

A credit derivative is a contract used to transfer the risk associated to bad credit from one party to another

- Became very popular in the early 2000s as a distinct asset class
- The most popular are credit default swaps (CDS survived 2008)
- Honorable mention to collateralized debt obligations (CDO did not survived 2008).
- Other least-known examples are CDO^{2}s, CLOs, CMOs, MBSs, ABSs

Over the counter market

A product is said to be sold *over the counter*, OTC for short, when the transaction takes place between two parties without the intermediation of an exchange facility

A product is sold on the market when a third party, usually an exchange, is employed by the seller to find a buyer or by the buyer to find a seller

- A steak bought at the butcher shop is an OTC transaction
- A phone bought from an auction, e.g. using e-bay, is a market transaction

Brief history of credit derivatives

- The first known transaction was in 1995 by JP Morgan (group of Blythe Masters)
- By the end of 1999 the notional became of the order of hundreds of billions of dollars (100,000,000,000$)
- In the early 2000s ISDA standardizes contract default events
- By 2004 implied CDO correlation are quoted on the market
- By 2007 the market grew to tens/hundreds of trillions
- The crisis of 2008-2009 greatly reduced CDOs transaction, CDSs survived
- In March 2009 starts the transition to CDS exchanged on regulated markets
- In 2011 there is still mixed market for CDSs

Default probabilities approaches

The pricing of credit derivatives is based on the ability to compute the risk-neutral probability of default at different future dates.

Two different approaches

- **Estimate** of default probabilities from the accounting analysis of future cash flows. Used, e.g., in capital requirements of Basel II (LGD models)

- **Compute** the market expectation of defaults, the default probability, from observed traded credit derivatives (described in this lecture)

Homogeneous Poisson process (1/2)

Also known as the bus-stop process:
The probability of a bus coming in the next minute does not depend on the bus schedule nor the current time: the bus will arrive late no matter what time it is or what the bus schedule days

The Poisson process $N(t)$ is a counting process such that

- Initially no events $N(0) = 0$. Increasing process $N(t + \Delta t) > N(t)$
- Increments are independent (events on non-overlapping intervals are independent)
- Increments are stationary (probability distribution on an interval depends only on the interval length)

Homogeneous Poisson process (2/2)

The homogeneous Poisson process is described by a single parameter h, the hazard rate, and that the probability of k events between t and $t + \Delta t$ is given by

$$P(k \text{ events at } t \in [t_1, t_1 + \Delta t]) = \frac{(h\,\Delta t)^k\, e^{-h\,\Delta t}}{k!}$$

It can be shown that no events can happen at the same time and that the average time between events is given by

$$\langle \Delta t \rangle = \tau = \frac{1}{h} \tag{16}$$

This is also the average time between now and the next event (Bus)

Poisson process and default events

In credit derivatives we want to compute the probability of no defaults between time t and $t + \Delta t$. Since

$$e^{-h\,\Delta t} + P(k \geq 1) = e^{-h\,\Delta t} + \sum_{k=1}^{\infty} \frac{(h\,\Delta t)^k\, e^{-h\,\Delta t}}{k!} = 1 \tag{17}$$

we have

$$P(k \geq 1) = 1 - e^{-h\,\Delta t} \tag{18}$$

so that

$$P^s(\Delta t|t) = P(k = 0) = 1 - P(k \geq 1) = e^{-h\,\Delta t} \tag{19}$$

The survival probability decreases like the exponential function

General shape of survival probability

The (cumulative) survival probability $P^s(t)$

- $P^s(t = 0)$=1
- Is always a non-increasing function of time
- Approaches zero as the time goes to infinity: for anybody, given enough time, the likelihood of default is certain

Hazard rate function

Market participants, in general, will have different expectations for the hazard rate h at different times. In general we will have a time-dependent hazard rate to give a survival probability of,

$$P^s(t) = \exp\left(-\int_0^t h(s)ds\right) . \qquad (20)$$

The simplest way to obtain a probability of default with a time-dependent hazard rate is to assume a piecewise-flat hazard rate.

Other approaches, such as piecewise-linear hazard rates, have been also proposed but not used in practice

Piecewise flat hazard rate (1/2)

Denoting with P^s the survival probability, consider three consecutive dates t_1, t_2, and t_3

$$P^s(t_1 < t \le t_3) = e^{-h\,(t_3 - t_1)} = e^{-h\,(t_3 - t_2)} e^{-h\,(t_2 - t_1)} \tag{21}$$

so that

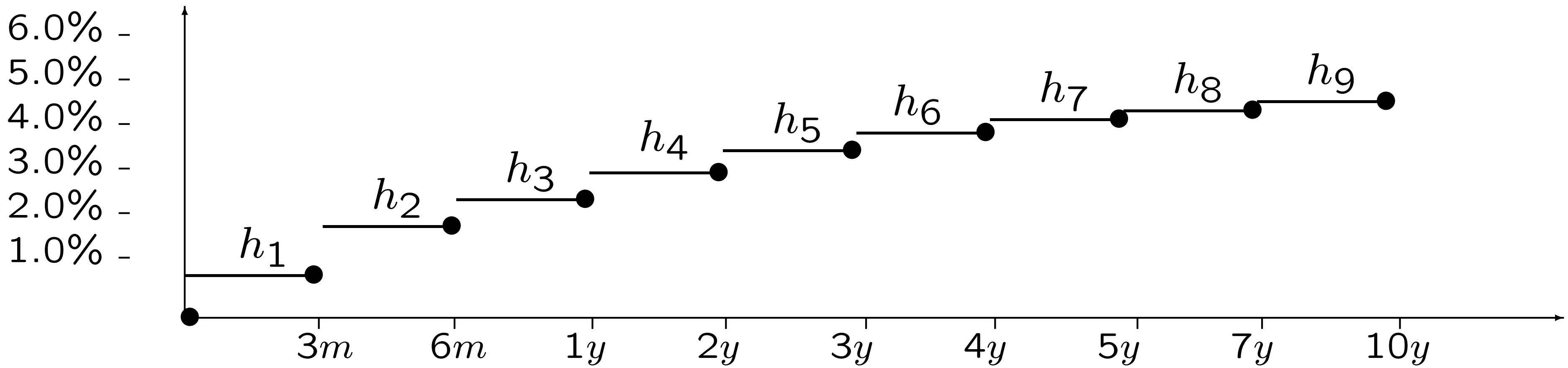
$$P^s(t_1 < t \le t_3) = P^s(t_1 < t \le t_2)\, P^s(t_2 < t \le t_3) \tag{22}$$

We can use this equation to assign different hazard rates to different forward periods.

Hazard rate h_1 is used between t_0=0 and t_1, h_2 is used between t_1 and t_2, and so on

Piecewise flat hazard rate (2/2)

Example of graph for a piecewise-flat hazard rate $h(t)$



Discount factor and survival probability

There is an analogy between discount factors and survival probabilities. Here we list the similarities

- Both have one as their present value
- They do not increase with time
- They are the building blocks of their markets (discount factor for the interest-rate market, survival probability for the credit market)
- The instantaneous hazard rate is the analogous of the instantaneous forward rate

Default probability

The default probability of a name is complementary to its survival probability

$$P(t) = 1 - P^s(t) \tag{23}$$

The probability of default, as standing at time t between two future successive dates t_1 and t_2, is given by

$$P(t_1 < \tau_{\text{default}} \leq t_2) = P(t_1, t_2) = P(t_2) - P(t_1)\,. \tag{24}$$

A similar equations holds for the survival probability

Credit default swaps (1/2)

A credit default swap, CDS for short, is a swap that provides insurance against missed payments from a financial entity (corporate, sovereign)

- The company is known as the CDS underlying *name*
- The event that triggers a CDS is known as the *credit event*
- There are two sides: the buyer and the seller of protection
- CDS spread, usually the five-year quote, are now a synonymous of the name credit worthiness
- When quoted on a market spreads are fixed and upfront are exchanged (quotes may still given as spreads)

Credit default swaps (2/2)

Given a CDS underlying name and a contract nominal

- The protection buyer makes periodic payments to the seller as a predetermined nominal percentage: the CDS spread
- The protection seller upon the trigger of a credit event, buys a bond, issued by the name, at face value from the protection seller
- If the credit event does not happen before maturity no payment is due from the seller to the buyer
- If the credit event does happen, after the due payment the CDS is extinguished

Recovery ratio

- One or more bonds issued by the target name is observed to determine the default event.
- In case of default the bond value at default is known as the *recovery value*
- The *recovery ratio*, R, is its ratio to the notional
- The difference between the bond face value and the bond market value multiplied by the notional is usually termed *loss given default* and is equal to $(1 - R)N$
- The exact value of the loss given default might not be known until few weeks after the actual default occurred

Standard CDS Contract Specification

see file `Standard.CDS.Contract.Specification.2009-05-12.pdf`

Questions?

Computation of CDS net present value

- Given a notional N
- an upfront percentage u paid at the settlement date T_s
- and a CDS spread s

The NPV of a credit default swap can be positive or negative:

$$\text{NPV} = N\left[u\,D(T_s) + s\,A + s\,B - C\right] \tag{25}$$

- $s\,N\,A$ is the coupon-payment leg
- $s\,N\,B$ is the accrual term
- $N\,C$ is the default leg

CDS fair spread

We will use the default probability curve and a numerical approximation to determine the coefficients A, B, and C

The credit-default swap fair spread is the value of $\tilde{s}$ that results in a null NPV

It can be shown that equation (25) implies

$$\tilde{s} = \frac{C - u\,D(T_s)}{A + B} \tag{26}$$

Example of quoted CDS fair spreads

Example of quoted fair CDS spreads for Senior MM Fiat CDS in currency EUR at three different dates. Spreads are quoted in basis points per year for a number of different maturities

Maturity	May 2006	July 2007	Nov. 2010
$3m$	22.4	19.6	81.90
$6m$	32.0	28.0	98.48
$1y$	40.0	35.0	109.82
$2y$	79.0	50.0	196.79
$3y$	118.0	65.0	266.14
$4y$	156.5	82.5	305.41
$5y$	195.0	100.0	344.71
$6y$	217.5	112.0	353.10
$7y$	240.0	124.0	361.77
$8y$	251.7	133.7	365.39
$9y$	263.3	143.3	369.15
$10y$	275.0	153.0	373.01
$11y$	275.2	153.2	372.68
$12y$	275.4	153.4	372.40
$15y$	276.0	154.0	371.85
$20y$	277.0	155.0	371.38
$30y$	278.0	156.0	371.09

Upfront payment

In order to make credit default swaps more standard, the same fixed spread is applied to many contracts: for example, 100 basis points for investment grade names and 400 basis points for all the others

In this case credit default swaps are quoted as cents over the dollar,

$$u = \frac{\mathrm{NPV}}{N} \tag{27}$$

or as a 100 basis

$$q = 100(1 - \tilde{s}) \tag{28}$$

The mid-point approximation (1/2)

- Coupons are always paid at coupon dates
- Defaults can only occur exactly half way through a coupon

It can be shown that A, B, and C, can be computed as

$$A = \sum_{i=1}^{n} [1 - P(t_i)]\, D(t_i) Y\left(t_{i-1}, t_i\right)\,, \tag{29}$$

$$B = -\sum_{i=1}^{n} P(t_{i-1}, t_i) D(t_{i-1/2}) Y\left(t_{i-1/2}, t_i\right)\,, \tag{30}$$

$$C = \sum_{i=1}^{n} (1 - R)\, P(t_{i-1}, t_i) D(t_{i-1/2})\,, \tag{31}$$

The mid-point approximation (2/2)

- t_0 is the beginning date of next coupon
- $t_1, \ldots, t_n$ are the coupon payment dates
- $t_{i+1/2}$ denoting the half point between t_i and t_{i+1}
- $D(t)$ is the risk-free discount factor at time t
- $Y(t, s)$ is the year fraction between times t and s

Definition of default event

CDS market did not fully evolve until a unified definition of default event was found. Default events are

- bankruptcy
- failure to pay a loan
- debt restructuring

Restructuring types (1/2)

Full Restructuring (FR)
This 1999 ISDA definition dictates that any restructuring event qualifies as a credit event. Therefore, even a restructuring that increases the value of present and future coupons can trigger the default event.

Modified Restructuring (MR)
This 2001 ISDA definition states that a credit event is triggered by all the restructuring agreements that do not cause a loss. While restructuring agreements still trigger a credit event, this clause limits the deliverable obligations to those with a maturity of 30 months or less after the termination date of the CDS.

Restructuring types (2/2)

Modified Modified Restructuring (MM)
In 2003 ISDA introduced a new definition of restructuring, similar to Modified Restructuring, where agreements still trigger credit events, but the remaining maturity of deliverable assets must be shorter than 60 months for restructured obligations and 30 months for all other obligations.

No Restructuring (NR)
Under this contract condition all restructuring events are excluded as trigger events. Some of the most popular CDS indexes in North America, including those belonging to the CDX index, are traded under this definition.

Questions?

Bootstrapping the probability term structure

We want the probability of default to be risk neutral and to match market expectations

We consider a number of CDS contracts referred to a certain issuer for a given restructuring type and seniority

Bootstrapping the probability term structure means finding a number of time nodes t_i's and hazard rates h_i's so that the quoted credit default swaps by the market are exactly re-priced

Probability term structure until first node

Set $t_0 = 0$ as the current time, t_1 as the maturity of the first quoted CDS, the variable h_1 is to be determined. Assume $D(T)$ is given. The expression for the probability of default $P(t)$ becomes

$$P(t) = 1 - e^{-h_1 t} \qquad \text{for} \quad t_0 \leq t \leq t_1 \,. \tag{32}$$

Note that $P(t)$ is not defined for $t > t_1$.
Compute $P(t_1)$ as

$$P(t_1) = 1 - e^{-h_1 t_1} \,. \tag{33}$$

Using the mid-point approximation we compute the first CDS NPV

$$\text{NPV}(\text{CDS}_1) = f(h_1) = 0 \tag{34}$$

and solve for h_1

Probability term structure until second node

Set t_2 as the maturity of the second quoted CDS, the variable h_2 is to be determined. Extend the probability structure $P(t)$

$$P(t) = 1 - [1 - P(t_1)]\, e^{-h_2(t-t_1)} \quad \text{for} \quad t_1 < t \leq t_2\,. \tag{35}$$

Compute

$$P(t_2) \;=\; 1 - e^{-h_1\, t_1} e^{-h_2\,(t_2-t_1)}$$

Since h_1 is known, compute the NPV of CDS_2 in terms of h_2

$$\text{NPV}(\text{CDS}_2) = f(h_2) = 0 \tag{36}$$

and solve for h_2

Probability term structure other nodes

Given the partial hazard rates $h_1, h_2, \ldots, h_{i-1}$, the probability term structure is known up to t_{i-1} included. Define t_i and

$$P(t) = 1 - [1 - P(t_{i-1})]\, e^{-h_i(t - t_{i-1})} \quad \text{for} \quad t_{i-1} < t \le t_i \tag{37}$$

Compute the NPV of the i-th CDS

$$\text{NPV}(\text{CDS}_i) = f(h_i) = 0 \tag{38}$$

and solve for h_i

This procedure can be continued until the last CDS is priced, obtaining an expression for the probability structure up to the maturity of the last CDS so that all the CDS spreads are matched

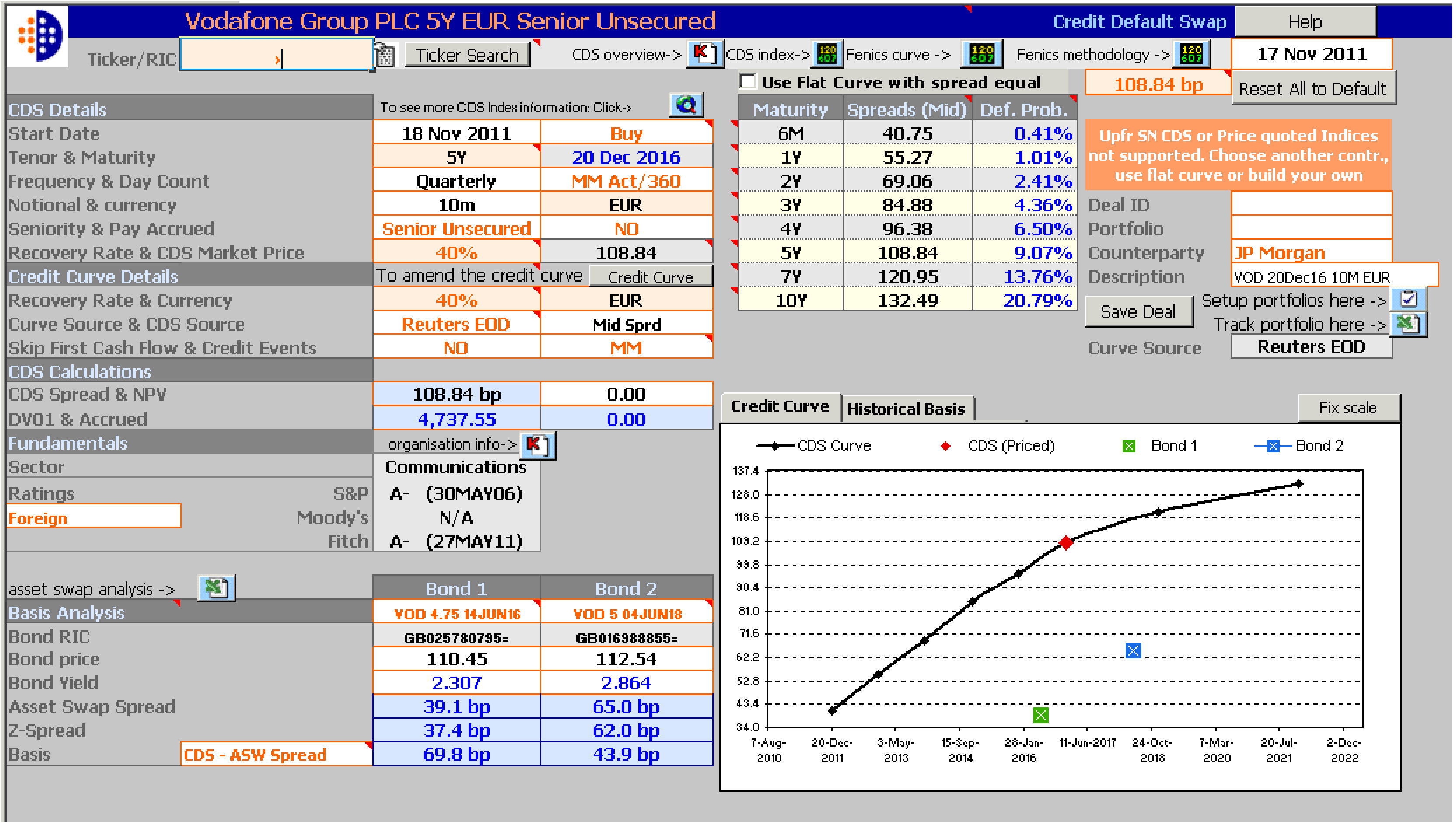
Extrapolation of probability term structure

Sometimes it is necessary to estimate probability of default after the latest quoted CDS. In these cases we extend the latest hazard rate to infinity:

$$P(t) = 1 - [1 - P(t_n)]\, e^{-h_n(t - t_n)} \qquad \text{for} \qquad t \geq t_n \qquad (39)$$

The knowledge of the default-probability term structure is used in pricing financial products while taking into account the credit worthiness of their issuers

Sample compuation of CDS from data provider



Questions?

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